prob.E Expectation

Recall that a random variable is a function $X : \Omega \to \mathbb{R}$ that maps from the sample space to the reals. Random variables are the arguments of probability mass functions (PMFs) and probability density functions (PDFs).
The **expected value** (or **expectation**) of a random variable is akin to its "average value" and depends on its PMF or PDF. The expected value of a random variable X is denoted $\langle X \rangle$ or $E[X]$. There are two definitions of the expectation, one for a discrete random variable, the other for a continuous random variable. Before we define, them, however, it is useful to predefine the most fundamental property of a random variable, its **mean**.

Definition prob.1: mean

The *mean* of a random variable X is defined as

$$m_X = E[X].$$

Let's begin with a discrete random variable.

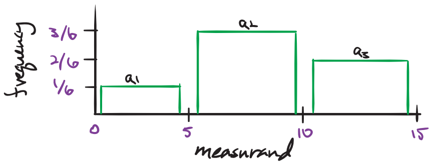
Definition prob.2: expectation of a discrete random variable

Let K be a discrete random variable and f its PMF. The *expected value* of K is defined as

$$E[K] = \sum_{\forall k} k f(k).$$

Example prob.E-1 re: expectation of a discrete random variable

Given a discrete random variable K with PMF shown below, what is its mean m_K?



Let us now turn to the expectation of a continuous random variable.

Definition prob.3: expectation of a continuous random variable

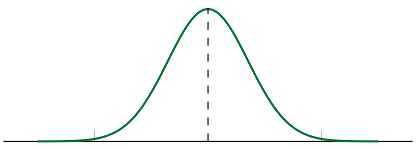
Let X be a continuous random variable and f its PDF. The *expected value* of X is defined as

$$\mathrm{E}\left[X\right] = \int_{-\infty}^{\infty} x f(x) dx.$$

Example prob.E-2

Given a continuous random variable X with Gaussian PDF f, what is the expected value of X?

re: expectation of a continuous random variable



Due to its sum or integral form, the expected value $\mathrm{E}[\cdot]$ has some familiar properties for random variables X and Y and reals a and b.

$$\mathrm{E}[a] = a \tag{1a}$$

$$\mathrm{E}[X + a] = \mathrm{E}[X] + a \tag{1b}$$

$$\mathrm{E}[aX] = a\,\mathrm{E}[X] \tag{1c}$$

$$\mathrm{E}[\mathrm{E}[X]] = \mathrm{E}[X] \tag{1d}$$

$$\mathrm{E}[aX + bY] = a\,\mathrm{E}[X] + b\,\mathrm{E}[Y]\,. \tag{1e}$$