

stats.exe Exercises for Chapter stats

Exercise stats.brew

You need to know the duration of time a certain stage of a brewing process takes. You set up an automated test environment that repeats the test 100 times, recorded in the following JSON¹ data file:

http://ricopic.one/mathematical_foundations/source/brew.json

Perform the following analysis.

- Download and parse the JSON file (it contains a single array).
- Estimate the duration of the process from the sample.
- Choose and justify an assumed probability density function for the random variable duration.
- Use this PDF model to compute a 99 percent confidence interval for your duration estimate.
- Compute your duration confidence interval for the range of confidence values [85, 99.99] percent.²
- Plot the confidence intervals over the range of confidence in said intervals.

———/20 p.

1. JSON is a simple and common programming language-independent data format. For parsing it with Matlab, see [jsondecode](https://www.mathworks.com/help/matlab/ref/jsondecode.html) here: [mathworks.com/help/matlab/ref/jsondecode.html](https://www.mathworks.com/help/matlab/ref/jsondecode.html). For parsing it with Python, see the module `json` here: docs.python.org/library/json.

2. Consider using a z- or t-score inverse CDF lookup function like `t.ppf` from `scipy.stats`.

Exercise stats.laboritorium

Use linear regression techniques to find the values of a , b , c , and d , in a cubic function of the form,

$$f(x) = ax^3 + bx^2 + cx + d,$$

using the data below.

x	f(x)
-2.0	-4.7
-1.5	-1.9
-1.0	1.5
-0.5	1.5
0.0	1.4
0.6	0.3
1.1	-1.5
1.6	0.0
2.1	0.6
2.6	4.2

Exercise stats.robotization

Use linear regression techniques to find the value of τ in the function,

$$f(t) = 1 - e^{-\frac{t^2}{\tau}}$$

Using the data below.

t	f(t)
0.1	0.02
0.6	0.34
1.1	0.74
1.6	0.94
2.1	0.98

Exercise stats.tired

There are 7 students enrolled in MME 502. If every week 5 students comes to class, for how many weeks could a unique set of 5 students come to class?

_____ / 10 p.

Exercise stats.strange

Use linear regression techniques to find ω and ϕ in the function, $f(t) = \sin(\omega t + \phi)$ using the data below.

_____ / 25 p.

t	f(t)
0.0	0.53
0.1	0.73
0.2	0.91
0.5	0.92
0.6	0.83
0.7	0.65
0.8	0.42
0.9	0.15
1.0	-0.1
1.1	-0.35
1.2	-0.58
1.3	-0.82
1.4	-0.91
1.7	-0.86
1.8	-0.76
1.9	-0.54

Note: there are an infinite number of solutions to the inverse sine function, $\sin^{-1}(x) = \pm y + n\pi$ where $n \in \mathbb{Z}$. You will have to utilize this definition to get your data in a linear form for fitting.

Vector calculus

A great many physical situations of interest to engineers can be described by **calculus**. It can describe how quantities continuously change over (say) time and gives tools for computing other quantities. We assume familiarity with the fundamentals of calculus: **limit**, **series**, **derivative**, and **integral**. From these and a basic grasp of vectors, we will outline some of the highlights of **vector calculus**. Vector calculus is particularly useful for describing the physics of, for instance, the following.

mechanics of particles wherein is studied the motion of particles and the forcing causes thereof

rigid-body mechanics wherein is studied the motion, rotational and translational, and its forcing causes, of bodies considered rigid (undeformable)

solid mechanics wherein is studied the motion and deformation, and their forcing causes, of continuous solid bodies (those that retain a specific resting shape)

fluid mechanics wherein is studied the motion and its forcing causes of fluids (liquids, gases, plasmas)

heat transfer wherein is studied the movement of thermal energy through and among bodies

electromagnetism wherein is studied the motion and its forcing causes of electrically charged particles

This last example was in fact very influential in the original development of both vector calculus and **complex analysis**.¹ It is not an exaggeration to say that the topics above comprise the majority of physical topics of interest in engineering.

A good introduction to vector calculus is given by Kreyszig (2011, Chapters 9, 10). Perhaps the most famous and enjoyable treatment is given by Schey (2005) in the adorably titled *Div, Grad, Curl and All that*.

It is important to note that in much of what follows, we will describe (typically the three-dimensional space of our lived experience) as a **euclidean vector space**: an n -dimensional vector space isomorphic to \mathbb{R}^n . As we know from linear algebra, any vector $\mathbf{v} \in \mathbb{R}^n$ can be expressed in any number of **bases**. That is, the vector \mathbf{v} is a basis-free object with multiple basis representations. The **components** and **basis vectors** of a vector change with basis changes, but the vector itself is **invariant**. A **coordinate system** is in fact *just a basis*. We are most familiar, of course, with **Cartesian coordinates**, which is the specific orthonormal basis \mathbf{b} for \mathbb{R}^n :

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \quad \dots, \quad \mathbf{b}_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \quad (1)$$

Manifolds are spaces that appear locally as \mathbb{R}^n , but can be globally rather different and can describe **non-euclidean geometry** wherein euclidean geometry's **parallel postulate** is invalid. Calculus on manifolds is the focus of **differential geometry**, a subset of which we can consider our current study. A motivation for further study of differential geometry is that it is very convenient when dealing with advanced applications of mechanics, such as rigid-body mechanics of robots and vehicles. A very nice

1. For an introduction to complex analysis, see Kreyszig (2011, Part D).

mathematical introduction is given by Lee (2012) and Bullo **and** Lewis (2005a) give a compact presentation in the context of robotics. Vector fields have several important properties of interest we'll explore in this chapter. Our goal is to gain an intuition of these properties and be able to perform basic calculation.