vecs.exe Exercises for Chapter vecs

Exercise vecs.light

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Consider a vector field $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ defined in Cartesian coordinates (x, y, z) as

$$\mathbf{F} = [x^2 - y^2, y^2 - z^2, z^2 - x^2]. \quad (1)$$

a. Compute the divergence of $\mathbf{F}$.
b. Compute the curl of $\mathbf{F}$.
c. Prove that, in a simply connected region of $\mathbf{R}^3$, line integrals of $\mathbf{F}$ are path-dependent.
d. Prove that $\mathbf{F}$ is *not* the gradient of a potential (scalar) function (i.e. that it does not have gradience, as we've called it).

Exercise vecs.hike

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The altitude of (x, y) points on a nearby mountain are modeled on the domain $-2 \leqslant x \leqslant 2$, $-2 \leqslant y \leqslant 2$ as,

$$f(x, y) = 2 - \frac{x^2}{4} + \cos(\frac{\pi}{2} y).$$

Using this model of the mountain:

1. Find the 3 dimensional path you would travel on if you were to start from the trailhead at $(x, y) = (-1, -1.5)$ and head in a straight line to the top of the mountain at $(0, 0)$.
2. Given the definition of work $W = \int_C F(r) \cdot dr$, write the equation for $F(r)$ from the acceleration of gravity and assuming a mass of $50\ \mathrm{kg}$.
3. Solve for the work to climb the mountain on your path from part **a**.
4. Once you get to the trailhead your friend wants to take a different route that they think will take less work. Prove that it takes the same amount of work, no matter what route you take to the top of the mountain.

5. On your way up the mountain you notice you have altitude sickness at location $(-1, -0.75)$ and need to get to a lower altitude as quickly as possible. What direction should you go to descend the fastest? Write your answer as a vector pointing in the direction you should go.

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Fourier and orthogonality