opt.exe Exercises for Chapter opt

Exercise opt.chortle

______/20 p.

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined as

$$f(\mathbf{x}) = \cos(x_1 - e^{x_2} + 2)\sin(x_1^2/4 - x_2^2/3 + 4) \quad (1)$$

Use the method of Barzilai **and** Borwein (1988) starting at $\mathbf{x}_0 = (1, 1)$ to find a minimum of the function.

Exercise opt.cummerbund

Consider the functions (a) $f_1 : \mathbb{R}^2 \to \mathbb{R}$ and (b) $f_2 : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f_1(\mathbf{x}) = 4(x_1 - 16)^2 + (x_2 + 64)^2 + x_1 \sin^2 x_1 \quad (2)$$

$$f_2(\mathbf{x}) = \frac{1}{2}\mathbf{x} \cdot A\mathbf{x} - \mathbf{b} \cdot \mathbf{x} \quad (3)$$

where

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 15 \end{bmatrix} \quad \text{and} \quad (4a)$$

$$b = \begin{bmatrix} -2 & 1 \end{bmatrix}^\top . \quad (4b)$$

Use the method of Barzilai **and** Borwein (**ibidem**) starting at some $\mathbf{x}_0$ to find a minimum of each function.

Exercise opt.melty

Maximize the objective function

$$f(\mathbf{x}) = \mathbf{c} \cdot \mathbf{x} \quad (5a)$$

for $\mathbf{x} \in \mathbb{R}^3$ and

$$\mathbf{c} = \begin{bmatrix} 3 & -8 & 1 \end{bmatrix}^\top \quad (5b)$$

subject to constraints

$$0 \leqslant x_1 \leqslant 20 \quad (6a)$$

$$-5 \leqslant x_2 \leqslant 0 \quad (6b)$$

$$5 \leqslant x_3 \leqslant 17 \quad (6c)$$

$$x_1 + 4x_2 \leqslant 50 \quad (6d)$$

$$2x_1 + x_3 \leqslant 43 \quad (6e)$$

$$-4x_1 + x_2 - 5x_3 \geqslant -99. \quad (6f)$$

Exercise opt.lateness

Using gradient decent find the minimum of the function,

$$f(x) = x_1^2 + x_2^2 - \frac{x_1}{10} + \cos(2x_1),$$

starting at the location $x = [-0.5,\ 0.75]^T$, and with a constant value $\alpha = 0.01$.

1. What is the location of the minimum you found?
2. Is this location the global minimum?

Nonlinear analysis

1 The ubiquity of near-linear systems and the tools we have for analyses thereof can sometimes give the impression that nonlinear systems are exotic or even downright flamboyant. However, a great many systems[1] important for a mechanical engineer are frequently hopelessly nonlinear. Here are a some examples of such systems.

- A robot arm.
- Viscous fluid flow (usually modelled by the navier-stokes equations).
- ______________________
- Anything that "fills up" or "saturates."
- Nonlinear optics.
- Einstein's field equations (gravitation in general relativity).
- Heat radiation and nonlinear heat conduction.
- Fracture mechanics.
- ______________________

1. As is customary, we frequently say "system" when we mean "mathematical system model." Recall that multiple models may be used for any given physical system, depending on what one wants to know.

2 Lest we think this is merely an inconvenience, we should keep in mind that it is actually the nonlinearity that makes many phenomena useful. For instance, the ____________ depends on the nonlinearity of its optics. Similarly, transistors and the digital circuits made thereby (including the microprocessor) wouldn't function if their physics were linear.

3 In this chapter, we will see some ways to formulate, characterize, and simulate nonlinear systems. Purely ________________________ are few for nonlinear systems. Most are beyond the scope of this text, but we describe a few, mostly in Lec. nlin.char. Simulation via numerical integration of nonlinear dynamical equations is the most accessible technique, so it is introduced.

4 We skip a discussion of linearization; of course, if this is an option, it is preferable. Instead, we focus on the _____________________.

5 For a good introduction to nonlinear dynamics, see Strogatz **and** Dichter (2016). A more engineer-oriented introduction is Kolk **and** Lerman (1993).