

# nlin.ss Nonlinear state-space models

1 A state-space model has the general form

$$\frac{dx}{dt} = f(x, u, t) \tag{1a}$$

$$y = \text{_____} \tag{1b}$$

where  $f$  and  $g$  are vector-valued functions that depend on the system. **Nonlinear state-space models** are those for which  $f$  is a \_\_\_\_\_ functional of either  $x$  or  $u$ . For instance, a state variable  $x_1$  might appear as  $x_1^2$  or two state variables might combine as  $x_1x_2$  or an input  $u_1$  might enter the equations as  $\log u_1$ .

## Autonomous and nonautonomous systems

2 An **autonomous system** is one for which  $f(x)$ , with neither time nor input appearing explicitly. A **nonautonomous system** is one for which either  $t$  or  $u$  do appear explicitly in  $f$ . It turns out that we can always write nonautonomous systems as autonomous by substituting in  $u(t)$  and introducing an extra \_\_\_\_\_ for  $t^2$ .

2. Strogatz and Dichter, 2016.

3 Therefore, without loss of generality, we will focus on ways of analyzing autonomous systems.

## Equilibrium

4 An **equilibrium state** (also called a \_\_\_\_\_)  $\bar{x}$  is one for which  $dx/dt = 0$ . In most cases, this occurs only when the input  $u$  is a constant  $\bar{u}$  and, for time-varying systems, at a given time  $\bar{t}$ . For autonomous systems, equilibrium occurs when the following holds:

$$\text{_____} \tag{2}$$

This is a system of nonlinear algebraic equations, which can be challenging to solve for

$\bar{x}$ . However, frequently, several solutions—that is, equilibrium states—do exist.