B.02 Fourier transforms

Table four.1 is a table with functions of time $f(t)$ on the left and corresponding Fourier transforms $F(\omega)$ on the right. Where applicable, T is the time-domain period, $\omega_0 2\pi/T$ is the corresponding angular frequency, $j = \sqrt{-1}$, $a \in \mathbb{R}^+$, and $b, t_0 \in \mathbb{R}$ are constants. Furthermore, f_e and f_0 are even and odd functions of time, respectively, and it can be shown that any function f can be written as the sum $f(t) = f_e(t) + f_0(t)$. **(Hsu1967)**

Table four.1: Fourier transform identities.

function of time t	function of frequency ω
$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
$f(at)$	$\frac{1}{\lvert a \rvert} F(\omega/a)$
$f(-t)$	$F(-\omega)$
$f(t - t_0)$	$F(\omega) e^{-j\omega t_0}$
$f(t) \cos \omega_0 t$	$\frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$
$f(t) \sin \omega_0 t$	$\frac{1}{j2} F(\omega - \omega_0) - \frac{1}{j2} F(\omega + \omega_0)$
$f_e(t)$	$\operatorname{Re} F(\omega)$
$f_0(t)$	$j \operatorname{Im} F(\omega)$
$F(t)$	$2\pi f(-\omega)$
$f'(t)$	$j\omega F(\omega)$
$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$

$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{1}{j\omega}F(\omega) + \pi F(0)\delta(\omega)$
$-jtf(t)$	$F'(\omega)$
$(-jt)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)d\tau$	$F_1(\omega)F_2(\omega)$
$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega) * F_2(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(\alpha)F_2(\omega-\alpha)d\alpha$
$e^{-at}u_s(t)$	$\frac{1}{j\omega + a}$
$e^{-a\vert t\vert}$	$\frac{2a}{a^2+\omega^2}$
e^{-at^2}	$\sqrt{\pi/a}\, e^{-\omega^2/(4a)}$
1 for $\vert t\vert < a/2$, else 0	$\frac{a\sin(a\omega/2)}{a\omega/2}$
$te^{-at}u_s(t)$	$\frac{1}{(a+j\omega)^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-at})^n u_s(t)$	$\frac{1}{(a+j\omega)^n}$
$\frac{1}{a^2+t^2}$	$\frac{\pi}{a}e^{-a\vert\omega\vert}$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
$u_s(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$u_s(t-t_0)$	$\pi\delta(\omega) + \frac{1}{j\omega}e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
t	$2\pi j\delta'(\omega)$
t^n	$2\pi j^n \frac{d^n\delta(\omega)}{d\omega^n}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$

$\cos\omega_0 t$	$\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
$\sin\omega_0 t$	$-j\pi\delta(\omega-\omega_0)+j\pi\delta(\omega+\omega_0)$
$u_s(t)\cos\omega_0 t$	$\dfrac{j\omega}{\omega_0^2-\omega^2}+\dfrac{\pi}{2}\delta(\omega-\omega_0)+\dfrac{\pi}{2}\delta(\omega+\omega_0)$
$u_s(t)\sin\omega_0 t$	$\dfrac{\omega_0}{\omega_0^2-\omega^2}+\dfrac{\pi}{2j}\delta(\omega-\omega_0)-\dfrac{\pi}{2j}\delta(\omega+\omega_0)$
$tu_s(t)$	$j\pi\delta'(\omega)-1/\omega^2$
$1/t$	$\pi j-2\pi j u_s(\omega)$
$1/t^n$	$\dfrac{(-j\omega)^{n-1}}{(n-1)!}\left(\pi j-2\pi j u_s(\omega)\right)$
$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$
$\displaystyle\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\displaystyle\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$

Mathematics reference