

## C.02 Trigonometry

### Triangle identities

With reference to the below figure, the *law of sines* is

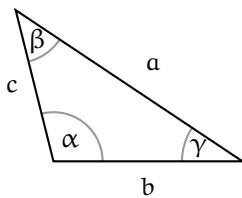
$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \quad (1)$$

and the *law of cosines* is

$$c^2 = a^2 + b^2 - 2ab \cos \gamma \quad (2a)$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta \quad (2b)$$

$$a^2 = c^2 + b^2 - 2cb \cos \alpha \quad (2c)$$



### Reciprocal identities

$$\csc u = \frac{1}{\sin u} \quad (3a)$$

$$\sec u = \frac{1}{\cos u} \quad (3b)$$

$$\cot u = \frac{1}{\tan u} \quad (3c)$$

### Pythagorean identities

$$1 = \sin^2 u + \cos^2 u \quad (4a)$$

$$\sec^2 u = 1 + \tan^2 u \quad (4b)$$

$$\csc^2 u = 1 + \cot^2 u \quad (4c)$$

**Co-function identities**

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \quad (5a)$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \quad (5b)$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \quad (5c)$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u \quad (5d)$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \quad (5e)$$

$$\cot\left(\frac{\pi}{2} - u\right) = \tan u \quad (5f)$$

**Even-odd identities**

$$\sin(-u) = -\sin u \quad (6a)$$

$$\cos(-u) = \cos u \quad (6b)$$

$$\tan(-u) = -\tan u \quad (6c)$$

**Sum-difference formulas (AM or lock-in)**

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v \quad (7a)$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v \quad (7b)$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \quad (7c)$$

**Double angle formulas**

$$\sin(2u) = 2 \sin u \cos u \quad (8a)$$

$$\cos(2u) = \cos^2 u - \sin^2 u \quad (8b)$$

$$= 2 \cos^2 u - 1 \quad (8c)$$

$$= 1 - 2 \sin^2 u \quad (8d)$$

$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u} \quad (8e)$$

**Power-reducing or half-angle formulas**

$$\sin^2 u = \frac{1 - \cos(2u)}{2} \quad (9a)$$

$$\cos^2 u = \frac{1 + \cos(2u)}{2} \quad (9b)$$

$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)} \quad (9c)$$

## Sum-to-product formulas

$$\sin u + \sin v = 2 \sin \frac{u+v}{2} \cos \frac{u-v}{2} \quad (10a)$$

$$\sin u - \sin v = 2 \cos \frac{u+v}{2} \sin \frac{u-v}{2} \quad (10b)$$

$$\cos u + \cos v = 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2} \quad (10c)$$

$$\cos u - \cos v = -2 \sin \frac{u+v}{2} \sin \frac{u-v}{2} \quad (10d)$$

## Product-to-sum formulas

$$\sin u \sin v = \frac{1}{2} [\cos(u-v) - \cos(u+v)] \quad (11a)$$

$$\cos u \cos v = \frac{1}{2} [\cos(u-v) + \cos(u+v)] \quad (11b)$$

$$\sin u \cos v = \frac{1}{2} [\sin(u+v) + \sin(u-v)] \quad (11c)$$

$$\cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)] \quad (11d)$$

## Two-to-one formulas

$$A \sin u + B \cos u = C \sin(u + \phi) \quad (12a)$$

$$= C \cos(u + \psi) \text{ where } (12b)$$

$$C = \sqrt{A^2 + B^2} \quad (12c)$$

$$\phi = \arctan \frac{B}{A} \quad (12d)$$

$$\psi = -\arctan \frac{A}{B} \quad (12e)$$