

D.01 Euler's formulas

Euler's formula is our bridge back-and forth between trigonometric forms ($\cos \theta$ and $\sin \theta$) and complex exponential form ($e^{j\theta}$):

$$e^{j\theta} = \cos \theta + j \sin \theta. \quad (1)$$

Here are a few useful identities implied by Euler's formula.

$$e^{-j\theta} = \cos \theta - j \sin \theta \quad (2a)$$

$$\cos \theta = \operatorname{Re}(e^{j\theta}) \quad (2b)$$

$$= \frac{1}{2}(e^{j\theta} + e^{-j\theta}) \quad (2c)$$

$$\sin \theta = \operatorname{Im}(e^{j\theta}) \quad (2d)$$

$$= \frac{1}{j2}(e^{j\theta} - e^{-j\theta}). \quad (2e)$$

Bibliography

- Ash, Robert B. (2008). *Basic Probability Theory*.
Dover Publications, Inc.
- Bagaria, Joan (2019). "Set Theory" in *The Stanford Encyclopedia of Philosophy*:
by editor Edward N. Zalta. Fall 2019.
Metaphysics Research Lab, Stanford
University.
- Baghramian, Maria and J. Adam Carter (2019).
"Relativism" in *The Stanford Encyclopedia of
Philosophy*: by editor Edward N. Zalta. Winter
2019. Metaphysics Research Lab, Stanford
University.
- Barker, Stephen and Mark Jago (2012). "Being
Positive About Negative Facts" in *Philosophy
and Phenomenological Research*: 85.1,
pages 117–138. doi:
10.1111/j.1933-1592.2010.00479.x.
- Barzilai, Jonathan and Jonathan M. Borwein
(January 1988). "Two-Point Step Size
Gradient Methods" in *IMA Journal of
Numerical Analysis*: 8.1, pages 141–148. issn:
0272-4979. doi: 10.1093/imanum/8.1.141. This
includes an innovative line search method.
- Biletzki, Anat and Anat Matar (2018). "Ludwig
Wittgenstein" in *The Stanford Encyclopedia of
Philosophy*: by editor Edward N. Zalta.
Summer 2018. Metaphysics Research Lab,
Stanford University. An introduction to
Wittgenstein and his thought.
- Bove, Antonio, F. (Ferruccio) Colombini and
Daniele Del Santo (2006). *Phase space analysis*

of partial differential equations. eng. Progress in nonlinear differential equations and their applications ; v. 69. Boston ; Berlin: Birkhäuser. isbn: 9780817645212.

Brogan, William L (1991). *Modern Control Theory*. Third. Prentice Hall.

Bullo, Francesco **and** Andrew D. Lewis (2005a). *Geometric control of mechanical systems: modeling, analysis, and design for simple mechanical control systems*.

by editor J.E. Marsden, L. Sirovich **and** M. Golubitsky. Springer.

— (January 2005b). *Supplementary Chapters for Geometric Control of Mechanical Systems*¹.

Choukchou-Braham, A. **and others** (2013).

Analysis and Control of Underactuated Mechanical Systems. SpringerLink : Bücher. Springer International Publishing. isbn: 9783319026367.

1. FB/ADL:04.

Ciesielski, K. (1997). *Set Theory for the Working Mathematician*. London Mathematical Society Student Texts. Cambridge University Press. isbn: 9780521594653. A readable introduction to set theory.

David, Marian (2016). ?The Correspondence Theory of Truth? **in** *The Stanford Encyclopedia of Philosophy*: **by editor** Edward N. Zalta. Fall 2016. Metaphysics Research Lab, Stanford University. A detailed overview of the correspondence theory of truth.

Dolby, David (2016). ?Wittgenstein on Truth? **in** *A Companion to Wittgenstein*: John Wiley & Sons, Ltd. **chapter 27**, **pages 433–442**. isbn: 9781118884607. doi:

10.1002/9781118884607.ch27.

Eccles, Peter J. (1997). *An Introduction to Mathematical Reasoning: Numbers, Sets and Functions*. Cambridge University Press. doi: 10.1017/CB09780511801136. A gentle introduction to mathematical reasoning. It

includes introductory treatments of set theory and number systems.

Enderton, H.B. (1977). *Elements of Set Theory*.

Elsevier Science. isbn: 9780080570426. A gentle introduction to set theory and mathematical reasoning—a great place to start.

Feynman, Richard P., Robert B. Leighton **and** Matthew Sands (2010). *The Feynman Lectures on Physics*. New Millennium. Perseus Basic Books.

Glanzberg, Michael (2018). **?Truth?** in *The Stanford Encyclopedia of Philosophy*: **by editor** Edward N. Zalta. Fall 2018. Metaphysics Research Lab, Stanford University.

Glock, Hans Johann (**january** 2006). **?Truth in the Tractatus?** in *Synthese*: 148.2, **pages** 345–368. issn: 1573-0964. doi: 10.1007/s11229-004-6226-2.

Gómez-Torrente, Mario (2019). **?Alfred Tarski?** in *The Stanford Encyclopedia of Philosophy*: **by editor** Edward N. Zalta. Spring 2019. Metaphysics Research Lab, Stanford University.

Guyer, Paul **and** Rolf-Peter Horstmann (2018). **?Idealism?** in *The Stanford Encyclopedia of Philosophy*: **by editor** Edward N. Zalta. Winter 2018. Metaphysics Research Lab, Stanford University.

Haberman, R. (2018). *Applied Partial Differential Equations with Fourier Series and Boundary Value Problems (Classic Version)*. Pearson Modern Classics for Advanced Mathematics. Pearson Education Canada. isbn: 9780134995434.

Hegel, G.W.F. **and** A.V. Miller (1998). *Phenomenology of Spirit*. Motilal Banarsidass. isbn: 9788120814738.

Hodges, Wilfrid (2018a). **?Model Theory?** in *The Stanford Encyclopedia of Philosophy*:

- byeditor**Edward N. Zalta. Fall 2018.
Metaphysics Research Lab, Stanford
University.
- Hodges, Wilfrid (2018b). ?Tarski’s Truth
Definitions? **in***The Stanford Encyclopedia of
Philosophy*: **byeditor**Edward N. Zalta. Fall
2018. Metaphysics Research Lab, Stanford
University.
- Hylton, Peter **and** Gary Kemp (2019). ?Willard
Van Orman Quine? **in***The Stanford
Encyclopedia of Philosophy*:
byeditorEdward N. Zalta. Spring 2019.
Metaphysics Research Lab, Stanford
University.
- Jaynes, E.T. **and**others (2003). *Probability Theory:
The Logic of Science*. Cambridge University
Press. isbn: 9780521592710. An excellent and
comprehensive introduction to probability
theory.
- Kant, I., P. Guyer **and** A.W. Wood (1999).
Critique of Pure Reason. The Cambridge
Edition of the Works of Immanuel Kant.
Cambridge University Press. isbn:
9781107268333.
- Kennedy, Juliette (2018). ?Kurt Gödel? **in***The
Stanford Encyclopedia of Philosophy*:
byeditorEdward N. Zalta. Winter 2018.
Metaphysics Research Lab, Stanford
University.
- Khrentzos, Drew (2016). ?Challenges to
Metaphysical Realism? **in***The Stanford
Encyclopedia of Philosophy*:
byeditorEdward N. Zalta. Winter 2016.
Metaphysics Research Lab, Stanford
University.
- Klein, Peter (2015). ?Skepticism? **in***The Stanford
Encyclopedia of Philosophy*:
byeditorEdward N. Zalta. Summer 2015.
Metaphysics Research Lab, Stanford
University.

Kline, M. (1982). *Mathematics: The Loss of Certainty*. A Galaxy book. Oxford University Press. isbn: 9780195030853. A detailed account of the “illogical” development of mathematics and an exposition of its therefore remarkable utility in describing the world.

Kolk, W. Richard **and** Robert A. Lerman (1993). *Nonlinear System Dynamics*. 1 **edition**. Springer US. isbn: 978-1-4684-6496-2.

Kreyszig, Erwin (2011). *Advanced Engineering Mathematics*. 10th. John Wiley & Sons, Limited. isbn: 9781119571094. The authoritative resource for engineering mathematics. It includes detailed accounts of probability, statistics, vector calculus, linear algebra, fourier analysis, ordinary and partial differential equations, and complex analysis. It also includes several other topics with varying degrees of depth. Overall, it is the best place to start when seeking mathematical guidance.

Lee, John M. (2012). *Introduction to Smooth Manifolds*. second. **volume** 218. Graduate Texts in Mathematics. Springer.

Legg, Catherine **and** Christopher Hookway (2019). *Pragmatism* **in** *The Stanford Encyclopedia of Philosophy*: **by** editor Edward N. Zalta. Spring 2019. Metaphysics Research Lab, Stanford University. An introductory article on the philosophical movement “pragmatism.” It includes an important clarification of the pragmatic slogan, “truth is the end of inquiry.”

Liberzon, Daniel (2012). *Calculus of Variations and Optimal Control Theory: A Concise Introduction*. Princeton University Press. isbn: 9780691151878.

Raatikainen, Panu (2018). *Gödel’s Incompleteness Theorems* **in** *The Stanford*

Encyclopedia of Philosophy:

byeditor Edward N. Zalta. Fall 2018.

Metaphysics Research Lab, Stanford University. A thorough and contemporary description of Gödel's incompleteness theorems, which have significant implications for the foundations and function of mathematics and mathematical truth.

Redding, Paul (2018). ?Georg Wilhelm Friedrich Hegel? **in** *The Stanford Encyclopedia of Philosophy*: **by**editor Edward N. Zalta. Summer 2018. Metaphysics Research Lab, Stanford University.

Schey, H.M. (2005). *Div, Grad, Curl, and All that: An Informal Text on Vector Calculus*. W.W. Norton. isbn: 9780393925166.

Shields, Christopher (2016). ?Aristotle? **in** *The Stanford Encyclopedia of Philosophy*: **by**editor Edward N. Zalta. Winter 2016. Metaphysics Research Lab, Stanford University.

Skiena, Steven S. (2001). *Calculated Bets: Computers, Gambling, and Mathematical Modeling to Win*. Outlooks. Cambridge University Press. doi: [10.1017/CB09780511547089](https://www3.cs.stonybrook.edu/~skiena/jaijai/excerpts/node12.html). This includes a lucid section on probability versus statistics, also available here: <https://www3.cs.stonybrook.edu/~skiena/jaijai/excerpts/node12.html>.

Smith, George (2008). ?Isaac Newton? **in** *The Stanford Encyclopedia of Philosophy*: **by**editor Edward N. Zalta. Fall 2008. Metaphysics Research Lab, Stanford University.

Stoljar, Daniel **and** Nic Damnjanovic (2014). ?The Deflationary Theory of Truth? **in** *The Stanford Encyclopedia of Philosophy*: **by**editor Edward N. Zalta. Fall 2014.

Metaphysics Research Lab, Stanford University.

- Strauss, W.A. (2007). *Partial Differential Equations: An Introduction*. Wiley. isbn: 9780470054567. A thorough and yet relatively compact introduction.
- Strogatz, S.H. and M. Dichter (2016). *Nonlinear Dynamics and Chaos*. Second. Studies in Nonlinearity. Avalon Publishing. isbn: 9780813350844.
- Textor, Mark (2016). ?States of Affairs? in *The Stanford Encyclopedia of Philosophy*: **by** editor Edward N. Zalta. Winter 2016. Metaphysics Research Lab, Stanford University.
- Virtanen, Pauli and others (july 2019). ?SciPy 1.0—Fundamental Algorithms for Scientific Computing in Python? in *arXiv e-prints*: arXiv:1907.10121, arXiv:1907.10121.
- Wikipedia (2019a). *Algebra* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=Algebra&oldid=920573802>. [Online; accessed 26-October-2019].
- (2019b). *Carl Friedrich Gauss* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=Carl%20Friedrich%20Gauss&oldid=922692291>. [Online; accessed 26-October-2019].
- (2019c). *Euclid* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=Euclid&oldid=923031048>. [Online; accessed 26-October-2019].
- (2019d). *First-order logic* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=First-order%20logic&oldid=921437906>. [Online; accessed 29-October-2019].
- (2019e). *Fundamental interaction* — *Wikipedia, The Free Encyclopedia*. [http://en.wikipedia.org/w/index.php?title=Fundamental%](http://en.wikipedia.org/w/index.php?title=Fundamental%20interaction)

- 20interaction&oldid=925884124. [Online; accessed 16-November-2019].
- Wikipedia (2019f). *Leonhard Euler* — *Wikipedia, The Free Encyclopedia*.
<http://en.wikipedia.org/w/index.php?title=Leonhard%20Euler&oldid=921824700>. [Online; accessed 26-October-2019].
- (2019g). *Linguistic turn* — *Wikipedia, The Free Encyclopedia*.
<http://en.wikipedia.org/w/index.php?title=Linguistic%20turn&oldid=922305269>. [Online; accessed 23-October-2019]. Hey, we all do it.
- (2019h). *Probability space* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=Probability%20space&oldid=914939789>. [Online; accessed 31-October-2019].
- (2019i). *Propositional calculus* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=Propositional%20calculus&oldid=914757384>. [Online; accessed 29-October-2019].
- (2019j). *Quaternion* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=Quaternion&oldid=920710557>. [Online; accessed 26-October-2019].
- (2019k). *Set-builder notation* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=Set-builder%20notation&oldid=917328223>. [Online; accessed 29-October-2019].
- (2019l). *William Rowan Hamilton* — *Wikipedia, The Free Encyclopedia*. <http://en.wikipedia.org/w/index.php?title=William%20Rowan%20Hamilton&oldid=923163451>. [Online; accessed 26-October-2019].
- Wittgenstein, L., P.M.S. Hacker and J. Schulte (2010). *Philosophical Investigations*. Wiley. isbn: 9781444307979.

- Wittgenstein, Ludwig (1922). *Tractatus Logico-Philosophicus*. by editor C., family i=C., given=K. Ogden, given i=. O. Project Gutenberg. International Library of Psychology Philosophy and Scientific Method. Kegan Paul, Trench, Trubner & Co., Ltd. A brilliant work on what is possible to express in language—and what is not. As Wittgenstein puts it, “What can be said at all can be said clearly; and whereof one cannot speak thereof one must be silent.”
- Žižek, Slavoj (2012). *Less Than Nothing: Hegel and the Shadow of Dialectical Materialism*. Verso. isbn: 9781844678976. This is one of the most interesting presentations of Hegel and Lacan by one of the most exciting contemporary philosophers.