Lecture 02.01 Types of signals

There are many types of signals that will be encountered in engineering analysis. This lecture surveys a few of the most common. Many of our signals are functions of *time* t, and are presented as such here, but it is possible to have other independent variables for a signal.

periodic **aperiodic**

The primary distinction that we make among types of signals is between those that are *periodic* and those that are *aperiodic*. Periodic signals repeat. In maths, a signal $v(t)$ is periodic if for all t and for some $T \in \mathbb{R}$,

$$v(t+T) = v(t). \tag{02.1}$$

If a function satisfies this condition, it is periodic with period T. An aperiodic function is one that is not periodic.

02.01.1 Sinusoidal signals

The familiar sine wave is the most popular periodic signal.

Equation 02.2 sinusoidal signal

amplitude **angular frequency** **phase** **cyclic frequency**

We call A the *amplitude*, ω the *angular frequency* in rad/s, and ϕ the *phase* in rad. We define the *cyclic frequency* f in Hz to be

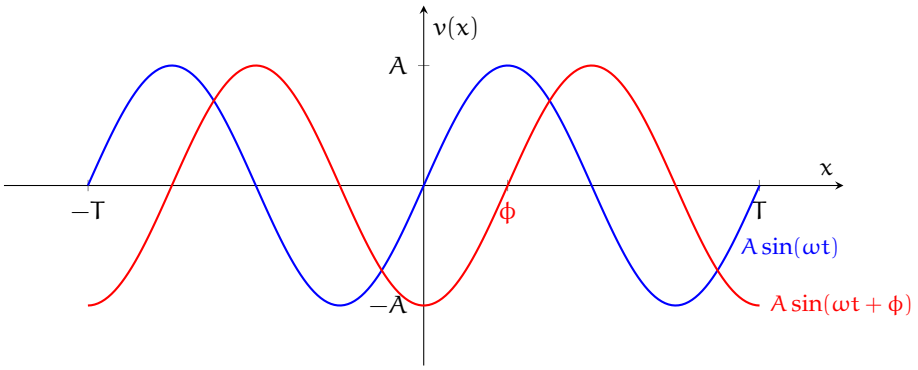
$$f = \frac{\omega}{2\pi} \tag{02.3}$$

period

and the *period* to be

$$T = \frac{1}{f}. \tag{02.4}$$

Below is a sinusoidal signal with and without a phase shift ϕ.



There are three common types of "amplitude" for sinusoids.

GOFA Good old-fashioned amplitude A is the amplitude we've described already.

P2P *Peak-to-peak amplitude* A_{pp} is twice the GOFA: $A_{pp} = 2A$. peak-to-peak amplitude

RMS *Root mean square* amplitude is conceptually like the "mean" amplitude but is defined over the time interval $[t_1, t_2]$ as root mean square amplitude

$$A_{rms} = \left(\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v(\tau)^2 d\tau \right)^{1/2}. \tag{02.5}$$

For a sinusoid, for time intervals that are multiples of the period, this reduces to

$$A_{rms} = A/\sqrt{2}. \tag{02.6}$$

One must be careful to specify which is being used and be aware that in some instances it is assumed to be "conventional."

Finally, a sinusoid's mean value is called the *dc offset* and is defined as the integral over a period divided by a period: dc offset

$$\bar{v} = \frac{1}{T} \int_{-T/2}^{T/2} v(t) dt. \tag{02.7}$$

Many signals are approximately sinusoidal. For instance, ac electrical power is sinusoidal, as is the motion of a pendulum.

02.01.2 Decibels

Although this is not a "type" of signal, it is worth mentioning decibels alongside sinusoids. The *decibel* (dB) is a conventional logarithmic ratio of amplitudes. decibel

Equation 02.8 decibel

The "reference" amplitude A_0 is sometimes taken to be 1, but typically it is some conventional quantity like 1 V for a voltage signal. In certain cases like this, the abbreviation dB might be given some decoration, like dBV.

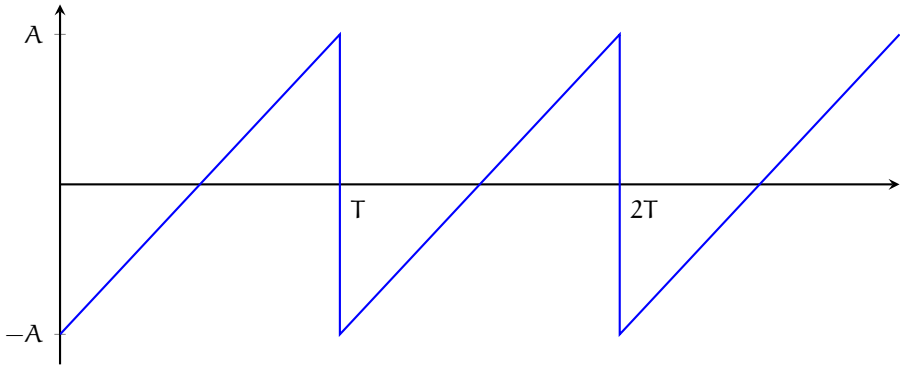
The decibel is sometimes expressed in terms of the ratio of the powers of two signals, but we aren't there yet.

02.01.3 Ramp and sawtooth

ramp
sawtooth

A *ramp* function is an aperiodic signal that increases linearly. A *sawtooth* function is a periodic signal that repeats a section of a ramp function.

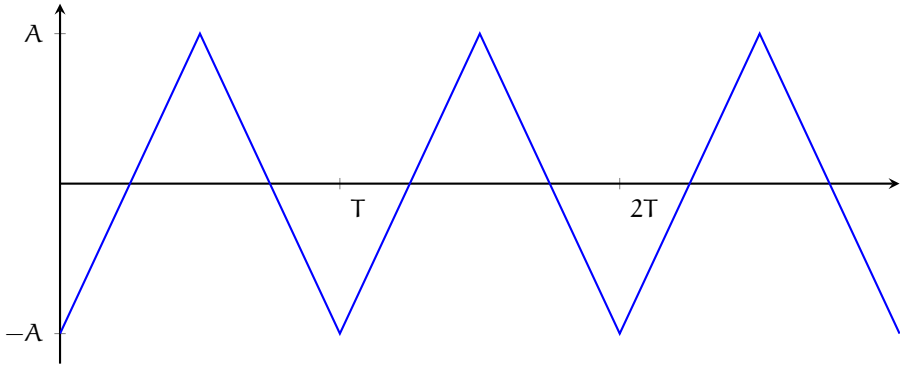
Below, a sawtooth signal with amplitude A and period T is plotted.



02.01.4 Triangle

A *triangle* function is a periodic signal with a section of a ramp followed by a negative section of the same ramp. **triangle**

Below, a triangle signal with amplitude A and period T is plotted.



02.01.5 Square

A *square* signal is a periodic signal that switches between two constant values. **square**

Below, a square signal with amplitude A and period T is plotted.