Lecture 04.02 Introduction to set theory

set theory

Set theory is a very useful branch of mathematics for engineers. In probability theory, we use the language of set theory. For this reason, we review basic set theory.

set

A *set* is a collection of objects. Set theory gives us a way to describe these collections. Often, the objects in a set are numbers or sets of numbers. However, a set could represent collections of zebras and trees and hairballs. For instance, here are some sets:

field **addition** **multiplication** **subtraction** **division** **real numbers** **set membership**

A *field* is a set with special structure. This structure is provided by the *addition* (+) and *multiplication* (×) operators and their inverses *subtraction* (−) and *division* (÷). The quintessential example of a field is the set of *real numbers* $\mathbb{R}$, which admits these operators, making it a field. The reals $\mathbb{R}$, the complex numbers $\mathbb{C}$, the integers $\mathbb{Z}$, and the natural numbers[2] $\mathbb{N}$ are the fields we typically consider.

Set membership is the belonging of an object to a set. It is denoted with the symbol $\in$, which can be read "is an element of," for element x and set X:

For instance, we might say $7 \in \{1, 7, 2\}$ or $4 \notin \{1, 7, 2\}$. Or, we might declare that a is a real number by stating: $x \in \mathbb{R}$.

set operations

Set operations can be used to construct new sets from established sets. We consider a few common set operations, now.

union

The *union* $\cup$ of sets is the set containing all the elements of the original sets (no repetition allowed). The union of sets A and B is denoted $A \cup B$. For instance, let $A = \{1, 2, 3\}$ and $B = \{-1, 3\}$; then

intersection

The *intersection* $\cap$ of sets is a set containing the elements common to all the original sets. The intersection of sets A and B is denoted $A \cap B$. For instance, let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$; then

[2]When the natural numbers include zero, we write $\mathbb{N}_0$.

empty set

If two sets have no elements in common, the intersection is the *empty set* $\varnothing = \{\}$, the unique set with no elements.

set difference

The *set difference* of two sets A and B is the set of elements in A that aren't also in B. It is denoted $A \setminus B$. For instance, let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Then

subset

proper subset

A *subset* $\subseteq$ of a set is a set, the elements of which are contained in the original set. If the two sets are equal, one is still considered a subset of the other. We call a subset that is not equal to the other set a *proper subset* $\subset$. For instance, let $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Then

complement

The *complement* of a subset is a set of elements of the original set that aren't in the subset. For instance, if $B \subseteq A$, then the complement of B, denoted $\overline{B}$ is