

## Lecture 04.03 Basic probability theory

The *sample space*  $\Omega$  of an experiment is the set representing all possible outcomes of the experiment. If a coin is flipped, the sample space is  $\Omega = \{H, T\}$ , where H is *heads* and T is *tails*. If a coin is flipped twice, the sample space could be

However, *the same experiment can have different sample spaces*. For instance, for two coin flips, we could also choose

We base our choice of  $\Omega$  on the problem at hand.

An *event* is a subset of the sample space. That is, an event corresponds to a yes-or-no question about the experiment. For instance, event A (remember:  $A \subseteq \Omega$ ) in the coin flipping experiment (two flips) might be  $A = \{HT, TH\}$ . A is an event that corresponds to the question, "Is the second flip different than the first?" A is the event for which the answer is "yes."

### 04.03.1 Algebra of events

Because events are sets, we can perform the usual set operations with them.

#### Example 04.03-1 set operations with events

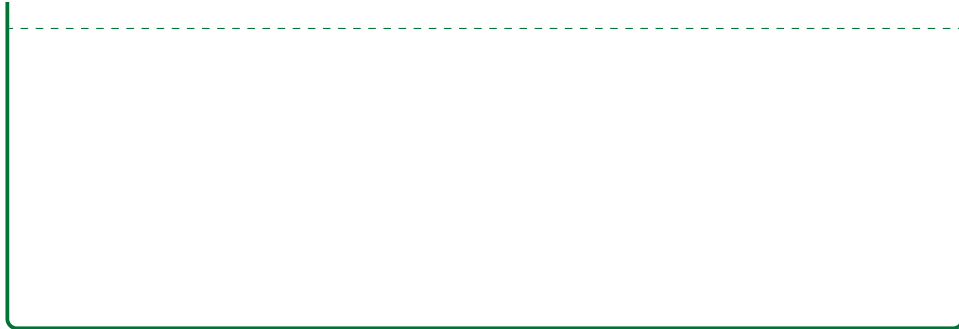
Consider a toss of a single die. We choose the sample space to be  $\Omega = \{1, 2, 3, 4, 5, 6\}$ . Let the following define events.

$$A \equiv \{\text{the result is even}\} = \{2, 4, 6\}$$

$$B \equiv \{\text{the result is greater than 2}\} = \{3, 4, 5, 6\}.$$

Find the following event combinations:

$$A \cup B \quad A \cap B \quad A \setminus B \quad B \setminus A \quad \bar{A} \setminus B.$$



**event class** The *event class*  $\mathcal{F}$  is often defined as the set of all subsets of  $\Omega$ . (It's actually more complicated, but we'll ignore that.) So  $\mathcal{F}$  is the set of all possible events given a sample space  $\Omega$ . When referring to an event, we often state that it is an element of  $\mathcal{F}$ . For instance, we might say an event  $A \in \mathcal{F}$ .

**probability measure** We're finally ready to assign probabilities to events. We define the *probability measure*  $P : \mathcal{F} \rightarrow [0, 1]$  to be a function satisfying the following conditions.

1. For every event  $A \in \mathcal{F}$ , the probability measure of  $A$  is greater than zero—i.e.  $P(A) \geq 0$ .
2. If an event is the entire sample space, its probability measure is unity—i.e. if  $A = \Omega$ ,  $P(A) = 1$ .
3. If events  $A_1, A_2, \dots$  are disjoint sets (no elements in common), then  $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$ .

**probability space** The three structures we've defined thus far— $\Omega$  (sample space),  $\mathcal{F}$  (event class), and  $P$  (probability measure)—are called the *probability space*  $(\Omega, \mathcal{F}, P)$ .

We conclude with the basics by observing four facts that can be proven from the definitions above.

- 1.
- 2.
- 3.
- 4.