## Lecture 04.03 Basic probability theory

**sample space**

The *sample space* $\Omega$ of an experiment is the set representing all possible outcomes of the experiment. If a coin is flipped, the sample space is $\Omega = \{H, T\}$, where H is *heads* and T is *tails*. If a coin is flipped twice, the sample space could be

However, *the same experiment can have different sample spaces*. For instance, for two coin flips, we could also choose

We base our choice of $\Omega$ on the problem at hand.

**event**

An *event* is a subset of the sample space. That is, an event corresponds to a yes-or-no question about the experiment. For instance, event $A$ (remember: $A \subseteq \Omega$) in the coin flipping experiment (two flips) might be $A = \{HT, TH\}$. $A$ is an event that corresponds to the question, "Is the second flip different than the first?" $A$ is the event for which the answer is "yes."

### 04.03.1 Algebra of events

Because events are sets, we can perform the usual set operations with them.

**Example 04.03-1 set operations with events**

Consider a toss of a single die. We choose the sample space to be $\Omega = \{1, 2, 3, 4, 5, 6\}$. Let the following define events.

$$A \equiv \{\text{the result is even}\} = \{2, 4, 6\}$$
$$B \equiv \{\text{the result is greater than } 2\} = \{3, 4, 5, 6\}.$$

Find the following event combinations:

$$A \cup B \qquad A \cap B \qquad A \setminus B \qquad B \setminus A \qquad \overline{A} \setminus B.$$

**event class**

The *event class* $\mathbb{F}$ is often defined as the set of all subsets of $\Omega$. (It's actually more complicated, but we'll ignore that.) So $\mathcal{F}$ is the set of all possible events given a sample sample space $\Omega$. When referring to an event, we often state that it is an element of $\mathcal{F}$. For instance, we might say an event $A \in \mathcal{F}$.

**probability measure**

We're finally ready to assign probabilities to events. We define the *probability measure* $P : \mathcal{F} \to [0, 1]$ to be a function satisfying the following conditions.

1. For every event $A \in \mathcal{F}$, the probability measure of $A$ is greater than zero—i.e. $P(A) \geqslant 0$.
2. If an event is the entire sample space, its probability measure is unity—i.e. if $A = \Omega$, $P(A) = 1$.
3. If events $A_1, A_2, \cdots$ are disjoint sets (no elements in common), then $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$.

**probability space**

The three structures we've defined thus far—$\Omega$ (sample space), $\mathcal{F}$ (event class), and $P$ (probability measure)—are called the *probability space* $(\Omega, \mathcal{F}, P)$.

We conclude with the basics by observing four facts that can be proven from the definitions above.

1.

2.

3.

4.