Lecture 04.04 Independence and conditional probability

Two events A and B are *independent* if and only if

independent

$$P(A \cap B) = P(A)P(B).$$

If an experimenter must make a judgment without data about the independence of events, she bases it on her knowledge of the events, as discussed in the following example.

Example 04.04-1 independence

Answer the following questions and imperatives.

1. Consider a single fair die rolled twice. What is the probability that both rolls are 6?
2. What changes if the die is biased by a weight such that $P(\{6\}) = 1/7$?
3. What changes if the die is biased by a magnet, rolled on a magnetic dice-rolling tray such that $P(\{6\}) = 1/7$?
4. What changes if there are two dice, biased by weights such that for each $P(\{6\}) = 1/7$, rolled once, both resulting in 6?
5. What changes if there are two dice, biased by magnets such that for each $P(\{6\}) = 1/7$, rolled once, both resulting in 6?

04.04.1 Conditional probability

dependent
conditional probability

If events A and B are somehow *dependent*, we need a way to compute the probability of B occurring given that A occurs. This is called the *conditional probability* of B given A, and is denoted $P(B|A)$. For $P(A) > 0$, it is defined as

$$P(B|A) = \frac{P(A \cap B)}{P(A)}. \tag{04.1}$$

We can interpret this as a restriction of the sample space Ω to A; i.e. the new sample space $\Omega' = A \subseteq \Omega$. Note that if A and B are independent, we obtain the obvious result:

Example 04.04-2 dependence

Consider two unbiased dice rolled once. Let events $A = \{\text{sum of faces} = 8\}$ and $B = \{\text{faces are equal}\}$. What is the probability the faces are equal given that their sum is 8?