Lecture 04.05 Bayes' theorem

Given two events A and B, *Bayes' theorem* (aka Bayes' rule) states that **Bayes' theorem**

$$P(A|B) = P(B|A)\frac{P(A)}{P(B)}. \qquad (04.2)$$

Sometimes this is written

This is a useful theorem for determining a test's effectiveness. If a test is performed to determine whether an event has occurred, we might as questions like "if the test indicates that the event has occurred, what is the probability it has actually occurred?" Bayes' theorem can help compute an answer.

The test can be either *positive* or *negative* and this result can be either *true* ☺ or *false* ☹.

There are four options, then. Consider an event A and an event that is a test result B indicating that event A has occurred. Table 04.1 shows these four possible test outcomes. Clearly, the desirable result for any test is that it is *true*. However, no test is true 100 percent of the time. So sometimes it is desirable to err on the side of the false positive, as in the case of a medical diagnostic. Other times, it is more desirable to err on the side of a false negative, as in the case of testing for defects in manufactured balloons (when a false negative isn't a big deal).

		A	$\overline{A}$
positive B		true ☺	false ☹
negative $\overline{B}$		false ☹	true ☺

Table 04.1: test outcome B for event A.

Some interesting results can be found from this. For instance, we can plot, as in Figure 04.1 the relationship between the probability of a positive test result given that the event actually occurs $P(B|A)$ and the probability of the event occurring given that the test is positive $P(A|B)$. (Note that, in both cases, it is the conditional probability of a true positive given some condition.)

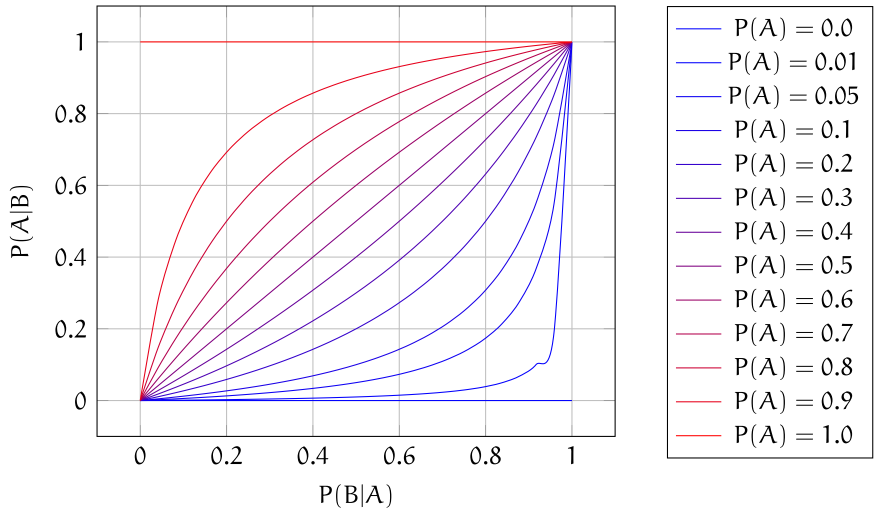

Figure 04.1: the probability that an event A occurred given that a test for it is positive B for different probabilities that the event A occurs.

Example 04.05-1 Bayes' theorem

Suppose 0.1% of springs manufactured at a given plant are defective. Suppose you need to design a test that has probability of 99% that a part, given that it is indicated defective, is actually so. What probability should your test have, given that a part is defective, of indicating that it is actually so?