Lecture 05.02 Functional propagation of uncertainty

Often, we use a measurement to estimate a quantity that is functionally dependent on the measurand. For instance, perhaps we would like to estimate the volume V of an object that—would you look at that—happens to be cubic with side length ℓ, so its volume could be reasonably estimated to be $V(\ell) = \ell^3$. Your measurement of ℓ has some associated uncertainty u_ℓ, certainly. How does that propagate to an uncertainty u_V in V?

Recall that uncertainty is half of a *symmetric* interval centered at the best estimate of the value. When you drop an interval symmetric about some value $\tilde{x}$ into a nonlinear function f, that interval comes out (usually) *asymmetric* about $\tilde{x}$.

asymmetry

Let's demonstrate this with our cubic volume. Let the 95% uncertainty in $\bar{\ell}$ be u_ℓ, such that there is a 95% probability that a volume measurement value

Now, run that interval $\bar{\ell} \pm u_\ell$ through the volume function V:

...this isn't symmetric about the mean $V(\bar{\ell})$ so we linearize ...which is what we also do for a multivariate function, too, and multiply each independent variable's partial derivative slope (evaluated at the mean) by the uncertainty of that variable's measurement. Then combine with RSS.