Lecture 06.02 Measuring resistance well

Many sensors are *resistive*, meaning the physical quantity to which they are sensitive affect the electrical resistance of the sensor, the accurate measurement of which is necessary for an accurate measurement of the physical quantity. In Lecture 06.01, we learned that we can measure unknown resistance R_u by applying a known voltage to it V_s, measuring the current i_{R_u} through it, and using Ohm's law:

resistive sensors

Furthermore, we learned an alternative is to place the unknown resistor in a voltage-divider circuit with a known resistor R_k, apply a known voltage V_s, measure the output voltage v_{R_k}, and use the voltage-divider equation

to solve for the unknown resistance

The sensitivity of these methods to measured quantities i_{R_u} and v_{R_k} are:

For small i_{R_u} or v_{R_k}, which correspond to large R_u, these are *very sensitive*. This means a small uncertainty in our measurements would propagate with large (and therefore unwanted) multiplicative factors.

We now explore the *Wheatstone bridge circuit* for measuring an unknown resistance.

Wheatstone bridge circuit

06.02.1 Wheatstone bridge circuit

A Wheatstone bridge circuit for measuring unknown resistance R_u from measured (known) V_s, v_o, R_1, R_2, and R_3 is shown in Figure 06.3. We would first like to derive the relationship between V_s and v_o. The first observation we make is that the two "arms" of the bridge, R_1–R_2 and R_3–R_u, are each just voltage dividers of V_s. That is,

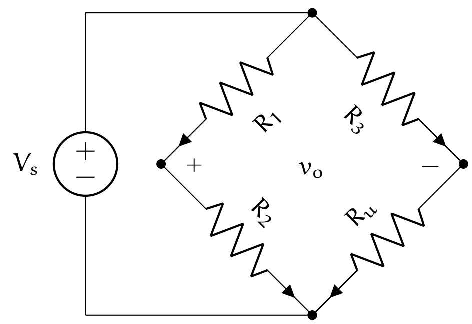

Figure 06.3: a Wheatstone bridge circuit.

By Kirchhoff's voltage law, $v_o = v_{R_2} - v_{R_u}$. These yield the desired relationship

$$v_o = \left(\frac{R_2}{R_1 + R_2} - \frac{R_u}{R_3 + R_u}\right) V_s. \tag{06.2}$$

Solving this for the unknown resistance, we obtain

$$R_u = \frac{R_3(R_2 V_s - (R_1 + R_2)v_o)}{R_1 V_s + (R_1 + R_2)v_o}. \tag{06.3}$$

It is typical common to have all resistors nearly equal to a single resistance R. Under this condition, the sensitivities of the measurement can be found to be

$$\frac{\partial R_u}{\partial v_o} = -\frac{4RV_s}{(V_s + 2v_o)^2},$$
$$\frac{\partial R_u}{\partial V_s} = \frac{4RV_s}{(V_s + 2v_o)^2}, \text{ and}$$
$$\frac{\partial R_u}{\partial R} = \frac{V_s - 2v_o}{V_s + 2v_o}.$$

In all these expressions, we can control our sensitivity with the input voltage V_s.

06.02.2 Null method

The bridge is said to be *balanced* when $V_s \neq 0$ and $v_o = 0$. That is, when

balanced bridge

From Equation 06.3, $v_o = 0$ greatly simplifies the expression for the unknown resistance

This is completely independent of V_s. Of course, if R_u is a resistive sensor, and its resistance changes such that the bridge is no longer balanced, the bridge must be re-balanced via changing another resistance some known amount. Often, R_2 is a *potentiometer* (variable resistor) that can be adjusted to balance the bridge. Sometimes feedback control is used to maintain a balanced bridge.

potentiometer

This is called the *null method* because it requires a balanced bridge (zero output voltage). It is difficult to measure a signal that is time-varying (unless it is slow) with this method, due to the required constant balancing of the bridge.

null method

06.02.3 Deflection method

The *deflection method* simply lets the bridge become unbalanced, logs the data, and applies Equation 06.3 to compute R_u. This is preferred for time-varying measurements, since it doesn't require a much faster bridge-balancing process. It does require that V_s is measured, which can, in some instances, lend a slight advantage to the null method in the case of stationary measurements.

deflection method